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INFLUENCE OF AN EMPIRICAL GEOLOGICAL STRENGTH INDEX METHOD FOR DETERMINING LINEAR AND NONLINEAR FAILURE CRITERIA

Angélica VIVANCO[1]*, Eslainer AVENDAÑO[1, 2]

[1] Mining Engineering, School of Engineering and Architecture, Arturo Prat University, Chile
[2] Department of Mining Engineering, Faculty of Engineering, University of Talca, Chile

Abstract: The construction of tunnels and underground galleries in mining has as fundamental input data the results of the failure criteria: traditionally Hoek–Brown and Mohr–Coulomb, to determine the failure envelopes that will allow the design of an economically exploitable mining system within viable safety frameworks that these criteria will guide. Therefore, the determination of rock mass resistance becomes fundamental and complex simultaneously due to the very nature of rock mass. Then, to identify a stressful state in which an excavation can be in conditions of stability it is necessary to have certain information both high in quality and economically valuable, which is not available in the early stages of the mining project. Thus, empirical methods and statistical relationships take notoriety, so this research evaluates the influence of an empirical method for the determination of the Geological Strength Index on the Mohr–Coulomb and Hoek–Brown failure criteria, with the benefit of estimating a stress field in which the excavation can self-sustain, evaluated in a first estimate in the pre-feasibility stage of the project, giving a guideline for design engineers. This research argues that the Geological Strength Index estimation method of Vivanco and Avendaño is recommended to estimate the Mohr–Coulomb failure criterion, but not the Hoek–Brown failure criterion.

Keywords**:** *failure criterion, Hoek–Brown, Mohr–Coulomb, GSI, Geological Strength Index*

1. INTRODUCTION

The construction of tunnels and underground galleries in mining has as fundamental input data the results of the failure criteria: traditionally Hoek–Brown (Hoek et al. 2002)

* Corresponding author: vivanco.avg@gmail.com (A. Vivanco)

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and Mohr–Coulomb (Coulomb 1773; Mohr 1900), to determine the failure envelopes that will allow designing an economically exploitable mining extraction system within viable safety frameworks that these criteria will guide. Similarly, determining failure criteria for slopes and hillsides is fundamental to ensure both the equipment, health, and life of people and the continued operability of mining projects and civil works.

Thus, determining the strength of the rock mass becomes fundamental and complex at the same time, due to the very nature of the rock mass: discontinuous, anisotropic, and heterogeneous, which makes it difficult to determine its behavior (Hussian et al. 2020; Yan et al. 2020; González De Vallejo et al. 2002; Ramírez and Alejano 2004). The resistance of a material can be defined as its capacity to absorb energy before undergoing changes that make it unusable for some purposes. Thus, it is possible to identify tensile strength, compressive strength, and resistance to deformation.

With all this information, it is possible to identify a stress state in which an excavation could be in a condition of stability for its self-sustainability, this is how the failure criteria are constructed. The failure criteria can be defined as a numerical method, of empirical origin, used to try to predict in an approximate way the behavior of a rock in failure (or of the rock mass), delimiting in graphical form the stress state in which the mining excavation will be safe and the maximum resistance that it will support, however, for its application numerous laboratory tests are required (Hussian et al. 2020). Among others, failure criteria in geomechanics, require some minimum input parameters to come to suggest a failure envelope that represents the governance of the stress states in an excavation, such parameters are: Deformation Modulus (E_{rm}); Disturbance Factor (D), which depends on the type of construction and state of it; Simple Compressive Strength, type of rock where the construction is carried out (parameter m_i); and Geological Strength Index (*GSI*) in quantitative terms. To obtain all this information it is necessary to have field equipment (to obtain the rock cores), and, in addition, to have laboratory equipment, of high economic value, and qualified personnel with expertise for its determination and interpretation, that is why, to be able to determine the failure criteria from a much more daily study would allow having a first approximation of the characteristics of the excavation that could be built and the cost that this could mean (Hassanpour et al. 2022; Sachpazis 1986; Rodríguez et al. 2018).

Given this premise, this research project is initiated, giving space to the study of empirical relationships that can provide a highly required parameter with a lot of variabilities in its results, such as the *GSI*. This index is usually very varied in its results because it is directly influenced by the observer's experience (Marinos et al. 2005; Marinos 2007; Morelli 2017; Wang and Aladejare 2016; Santa et al. 2019; Zhang 2016). The work proposed by Vivanco and Avendaño (2022) somehow remedies this problem since they propose to estimate a *GSI* value from the direct and exclusive measurement of *RQD%*, which is much more objective to determine. Thus, the authors recommend their methodology for when the *RQD%* value varies between 25% to 87% (Vivanco and Avendaño 2022).

Bieniawski clarifies that to correctly apply the Hoek–Brown failure criterion it is necessary to carefully use *GSI*, in an interval between 30 to 75 points (Bieniawski 2011), which is very similar to that determined by Vivanco and Avendaño for the use of *GSI* when estimated from *RQD%* which places its interval of use between 22 to 80 points, giving credence to the possibility of evaluating the failure criteria from this estimation.

This research aims to determine with what level of certainty the Hoek–Brown and Mohr–Coulomb failure criteria can be determined given an empirical estimate of *GSI*, with the benefit of estimating a safe stress field where the excavation can be self-supporting, evaluated in a first estimate at pre-feasibility stage.

2. LITERATURE REVIEW

2.1. FAILURE CRITERIA

In general, a failure criterion can be defined as a mathematical method used to try to predict approximately the deformation and failure behavior of the rock (or rock mass), there are two methodologies used to determine it, the first one is based on the stress state (the most used); and as a second option, there is the method based on the state of deformations (Xia et al. 2022; González De Vallejo et al. 2002; Eberhardt 2012; Hoek et al. 2002). In this way, the maximum resistance of the rock is determined by the stress that it can support.

Failure criteria are the basis of empirical methods and allow evaluating the strength of rock masses from the acting stresses and the properties of the rock material, providing: the response of intact rock to different stress conditions; prediction of the influence of discontinuities on the behavior of the rock mass; prediction of the global behavior of a rock mass (González De Vallejo et al. 2002; Hoek et al. 2002).

The stress state of a rock mass is determined by the magnitude and direction of its principal stresses, which will determine the behavior of the rock mass in terms of its deformation and eventual failure (Ros, 2008).

Of the most widely used methods, even to date, are the Mohr–Coulomb linear failure criterion, which is less appropriate for modeling the stability behavior of rock masses, but is very suitable for modeling the tensile behavior of soils, however, it is still used for its simplicity (González De Vallejo et al. 2002; Zhang and Salgado 2010; Sun et al. 2006; Bejarbaneh et al. 2015); and on the other hand, the Hoek–Brown nonlinear failure criterion, which better models the behavior of rock masses (González De Vallejo et al., 2002). There are other more recent failure criteria, however, they have not achieved the same diffusion and use as those of Mohr–Coulomb and Hoek–Brown (González De Vallejo et al. 2002).

2.1.1. MOHR–COULOMB FAILURE CRITERION

It was created by C.A. Coulomb in his publication of tests to problems of relative statics (Coulomb 1773) and later, by Otto Mohr, who at the end of the 19th century created a generalized form of the method (Mohr 1900; Yu 2002). The failure criterion is defined by a failure envelope (yield surface) which is constructed with a straight line that is tangent to a series of circles determined at failure of the material, it points out that failure occurs by a critical combination of normal and shear stresses according to Eq. (1) (Das and Sivakugan 2016). Through such circles, it is possible to determine the principal stresses.

The failure envelope is modeled by a straight line that is expressed in terms of the shear stress (τ) and normal stress (σ_N) and the angle of internal friction of the material (ϕ), as observed in equation (1) which is the most frequent mode (Fig. 1a), although, there is also a way to express it in terms of the principal stresses: major (σ_1); minor (σ_3); and uniaxial compressive stress (σ_c) as observed in equations (2) and (3) which is the less frequent mode (Fig. 1b). This criterion was initially created to model the behavior of soils, even so, and although the triaxial behavior of the rock does not fully coincide with the linear model, it is still widely used for rock massifs due to its simplicity and speed (Ros 2008; Budhu 2010; Gavilanes and Andrade 2004).

$$\tau = c + \sigma_N * \tan\phi, \tag{1}$$

$$\sigma_1 = \sigma_c + \sigma_3 * \tan\psi, \tag{2}$$

$$\tan\psi = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(\frac{\phi}{2} + \frac{\pi}{4}\right), \tag{3}$$

Below the Mohr–Coulomb envelope, there is a zone where the elastic stress states are considered safe states of stability, while the stress states above the envelope are in failure (Fig. 1). As a result of the application of this criterion, it is possible to obtain the cohesion and the angle of internal friction, the graphical representation of each of the Mohr circles, and the fitted line.

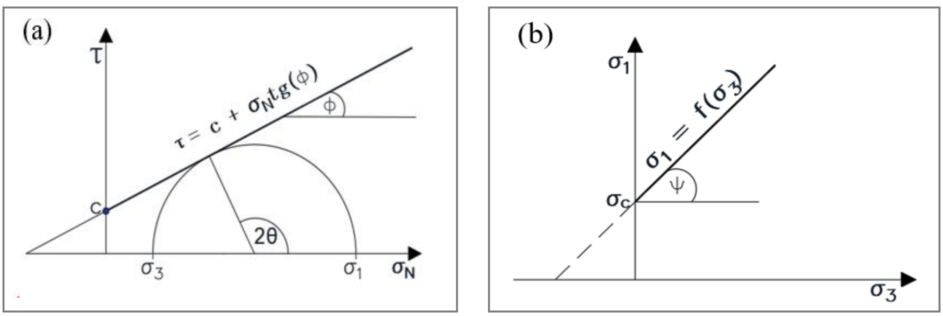


Fig. 1. Mohr–Coulomb failure criterion as a function of normal and tangential stresses: (a) principal stresses and (b) safe zone stress state where failure will not occur (González De Vallejo et al. 2002)

2.1.2. FAILURE CRITERION

This criterion is also known as Hoek and Brown's nonlinear failure criterion, of empirical origin and originally created in 1980 for the stability analysis of subway excavations in competent rock masses (Eberhardt 2012; Hoek and Marinos 2007). The graph describing the creep behavior corresponds to a curve (Eq. (4)), above which the stress state is "impossible" to reach since it represents a failure state; the area below this curve is considered safe, while the curve itself represents a failure stress state (Fig. 2).

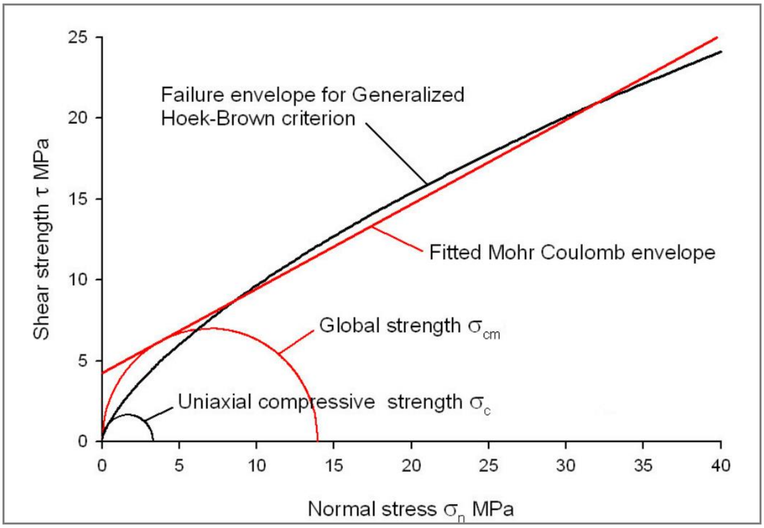


Fig. 2. Plot of the failure envelopes for the Generalized Hoek–Brown failure criterion and the associated Mohr–Coulomb criterion. In red line: the uniaxial compressive strength and the global strength of the rock mass are defined in this plot. Relationship between both criteria. Modified from Hoek, 2005

This criterion is expressed from equation (4), and in its generalized version is not exclusive of hard or competent rock masses. The value of m_i will depend on the type of material, and m_b is a value that will depend on this constant, Eq. (5); meanwhile, the constants *s* and *a* will depend on the geomechanical characteristics of the rock mass, mainly the Geological Strength Index or *GSI*, according to Eqs. (6) and (7); on the other hand, *D* is a value that varies according to the degree of alteration of the rock massif in which the subway excavation is to be carried out (Table 1) (Hoek et al. 2002).

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b * \frac{\sigma_s}{\sigma_{ci}} + s\right)^a, \tag{4}$$

$$m_b = m_i * \mathrm{e}^{\left(\frac{GSI-100}{28-14D}\right)}, \tag{5}$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)}, \tag{6}$$

$$a = \frac{1}{2} + \frac{1}{6}(e^{-GSI/15} - e^{-20/3}). \tag{7}$$

Although the Hoek–Brown failure criterion is widely accepted and used, determining the values of cohesion and friction is still a problem, according to the author himself. To solve it, in 2002, Hoek et al., made a reviewed of the failure criteria used in the mining field, analyzing them conceptually and mathematically, concluding that Rocscience's RocLab program (currently RSData), is a convenient and recommended method to solve the problems of cohesion, friction, and graph (Hoek et al. 2002).

With the generalized Hoek–Brown criterion, a best-fitting procedure in an artificial stress range to obtain equivalent Mohr–Coulomb parameters from the generalized Hoek–Brown criterion was also defined. Although other fitting methods exist, as noted by Saeidi et al. (2022): the method of Meng et al. (Meng et al. 2016) presents a uniform approximation procedure for elastic-plastic or elastic-fragile-plastic rock masses; and the method of Sofianos and Nomikos (2006) discussed two methods for supported and unsupported tunnels in elastic-plastic or elastic-fragile-plastic rocks (Saeidi et al. 2022), this research will use the fitting method presented in the RocScience software package, as it is the one recommended by Hoek, 2002 (Hoek et al. 2002).

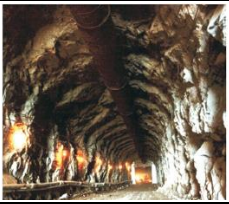
The RSData working software, from Rocscience, models both criteria and delivers as a result, in the case of the non-linear criterion, the graphical form of Tensile strength (σ_t); Uniaxial compressive strength (σ_c); Global strength (σ_{cm}); and the Modulus of deformation (E_{rm}) corresponding to the modulus of deformation of the rock mass (Hoek 2005).

Mohammadi (2015) compares both failure criteria, determining the maximum stresses in the failure plane, obtaining among other results, that both stresses and failure angles differ from the real ones, and that, in addition, the failure envelope affects the results obtained, being mostly influenced by it more than by the real failure plane (Mohammadi 2015).

Table 1. Guidelines for estimating disturbance factor *D* (Hoek et al. 2002)

Appearance of rock mass	Description of rock mass	Suggested value of D
1	**2**	**3**
	Excellent quality controlled blasting or excavation by Tunnel Boring Machine results in minimal disturbance to the confined rock mass surrounding a tunnel.	$D = 0$

Table 1 continued

1	2	3
	Mechanical or hand excavation in poor quality rock masses (no blasting) results in minimal disturbance to the surrounding rock mass.	$D = 0$
	Where squeezing problems result in significant floor heave, disturbance can be severe unless a temporary invert, as shown in the photograph, is placed.	$D = 0.5$ no invert
	Very poor quality blasting in a hard rock tunnel results in severe local damage, extending 2 or 3 m, in the surrounding rock mass.	$D = 0.8$
	Small scale blasting in civil engineering slopes results in modest rock mass damage, particularly if controlled blasting is used as shown on the left hand side of the photograph. However, stress relief results in some disturbance.	$D = 0.7$ good blasting $D = 1.0$ poor blasting
	Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal.	$D = 1.0$ production blasting
	In some softer rocks excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.	$D = 0.7$ mechanical excavation

2.2. GEOLOGICAL STRENGTH INDEX

The Geological Strength Index (*GSI*) is a rock mass classification system presented by Hoek and Brown in 1997, based on a simple and quick visual inspection performed by experienced professionals based on the geological conditions of the rock mass: at the level of structure, it considers the degree of alteration suffered by the rocks; the bond between them and their cohesion; and the surface aspects are alteration; erosion and/or type of texture; type of coating (Hoek and Brown 1997).

There are two major criticisms of this system: the first is the dependence of the result on the observer's experience (Marinos et al. 2007; Morelli 2017; Wang and Aladejare 2016; Santa et al. 2019; Zhang 2016); and the second, is that *GSI* was not determined from a known database.

2.3. GEOLOGICAL STRENGTH INDEX EMPIRICAL METHOD

Vivanco and Avendaño (2022) present a correlational analysis of the *GSI* and RQD% classification systems, using non-parametric statistics, with the objective of determining an expression that is capable of estimating *GSI* during field work from the minimum information that could be available at the beginning of the mining project. Among the results, it stands out that better *GSI* prediction results are obtained when 25% < *RQD*% ≤ 87%, improving the accuracy of the estimation presented by Santa et al. (2019) by 62% in that interval (Vivanco and Avendaño 2022).

Note the simplified equation (7) in which the percentage value of *RQD*% is used to estimate the value of *GSI*, denoted *GSI*′.

$$GSI' = 0.94 * RQD\% - 1.61. \tag{8}$$

Despite the above, it is still difficult to interpret with greater precision the specific geological characteristics of each rock mass (Vivanco and Avendaño 2022; Hassanpour et al. 2002)

The authors of the method recommend its use exclusively in the pre-feasibility phase of the mining project, when the information available is still scarce, and to be used as a general guideline of the possible behavior of the rock mass.

3. METHODOLOGY

The working methodology corresponds to the non-experimental quantitative type (Hernández 2018). The research is of an exploratory type, as it is an under-researched topic when limited field information is available, a complex and frequent situation in the early stages of a mining project (Zhang 2016; Huaman et al. 2017).

The direct deductive method is used to test the hypothesis stated in the research problem: is it possible to reliably estimate the and Mohr–Coulomb failure criterion from an empirical method of *GSI* estimation? To prove or disprove this hypothesis, a descriptive statistical study of the database was conducted to determine a comparative analysis of the parameters of interest in the Mohr–Coulomb and failure criterion, using as the only variation the estimation of the *GSI* value. The first *GSI* was determined by traditional method (Hoek and Brown 1997), then compared with the empirical equation of interest for this study (Vivanco and Avendaño 2022) to determine the influence of two empirical methods for the determination of linear (Mohr–Coulomb) and nonlinear (Hoek–Brown) failure criteria.

To carry out this study, 45 analysis cases are available >5000 m diamond holes drilled), the results of each failure criterion are compared with its simile determined with the *GSI* estimation of Vivanco and Avendaño (i.e., *GSI′*), and the errors are analyzed with different tolerances for the estimation error (±10%; ±15%; ±20%).

4. RESULTS OBTAINED

This procedure results in the case of the Mohr–Coulomb criterion, that: if a tolerance for error of ±10% is considered, then 38% of the cohesion estimates would be within this tolerance, as would 78% of the estimated friction angles; similarly, if a tolerance for error of ±15% is considered, then 51% of the cohesion estimates would be within this interval, as would 82% of the estimated friction angles; and, when considering a tolerance of ±20%, then the percentage of cohesion estimates that are within the indicated confidence interval increases, being 62%, as well as 87% of the estimated friction angles (Fig. 1). However, despite the dispersion of the data (Fig. 2), it is observed that the average of the errors committed is still low, determining an average error for the cohesion of –1.1% and an average error for the estimation of the angle of internal friction of the material of 1.6%. This shows that, even though the average of the estimates is low, there is a high variance of the estimates concerning the mean.

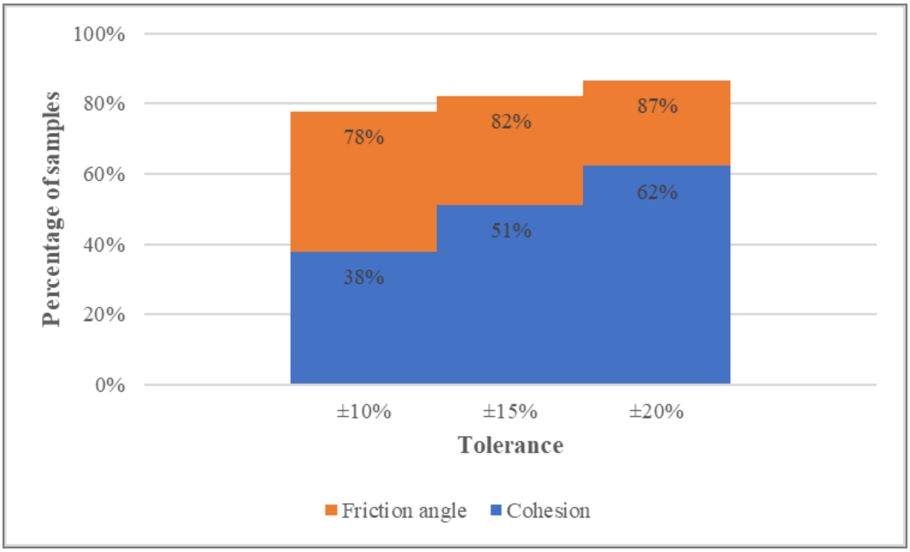


Fig. 3. Percentage amount of data as a function of a tolerance in the estimation error. Authors' own elaboration

However, the authors of *GSI′* recommend that estimates below 20 and above 80 points should be discarded for their estimation, so this recommendation is analyzed when determining the failure criteria. In this way, we work with 76% of the original data, obtaining for the Mohr–Coulomb criterion, that: if an error tolerance of ±10% is considered, then 35% of the cohesion estimates would be within this tolerance, as would 76% of the estimated friction angles; similarly, if an error tolerance of ±15% is considered, then 47% of the cohesion estimates would be within this range, as would 82% of the estimated friction angles; and, when considering a tolerance of ±20%, then the percentage of estimates that are within it increases to 56% for the case of cohesion, while 88% of the friction angles fall within this confidence interval (Fig. 3). However, despite the dispersion of the data (Fig. 4), it is observed that the average of the errors made is still low, determining an average error for the cohesion of –0.4% and an average error for the estimation of the angle of internal friction of the material of 2.4%. This shows that, even though there is a high variance of these errors concerning the mean, the average of the estimates is low.

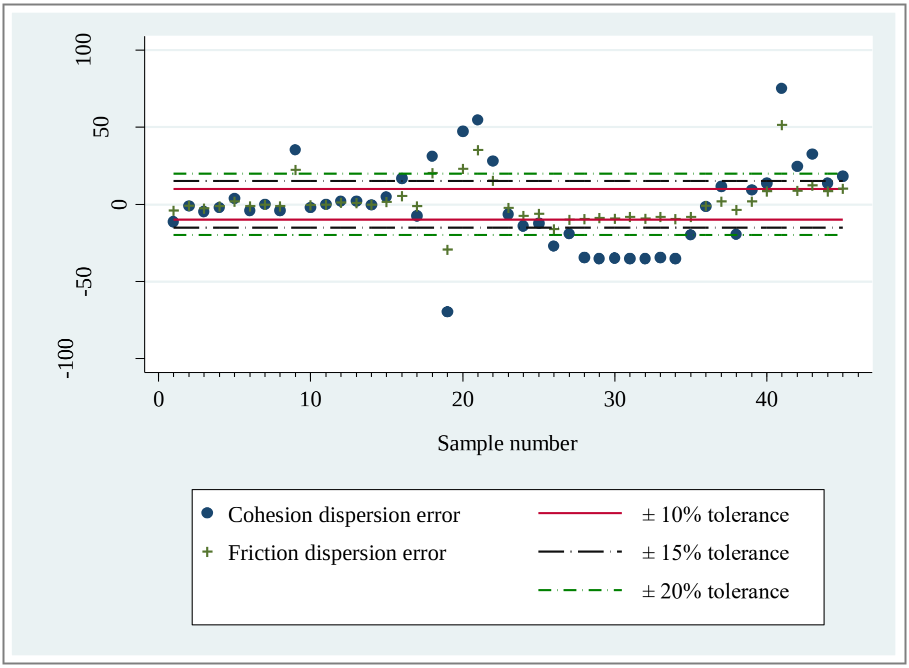


Fig. 4. Porcentual error of cohesion and friction angle. Authors’ own elaboration.
Note. Software Stata 14.2 Windows software. College Station, Texas 77845 USA: StataCorp.

It is observed that restricting the *GSI* estimates increases the dispersion, which could be attributed to the decrease in the amount of data processed. Then, although of showing good results to estimate the *GSI'* its application in the failure criterion does not present such good results, indicating that the error committed by the empirical method of *GSI* estimation accumulates and grows in a nonlinear way when it is used in the determination of the failure criteria.

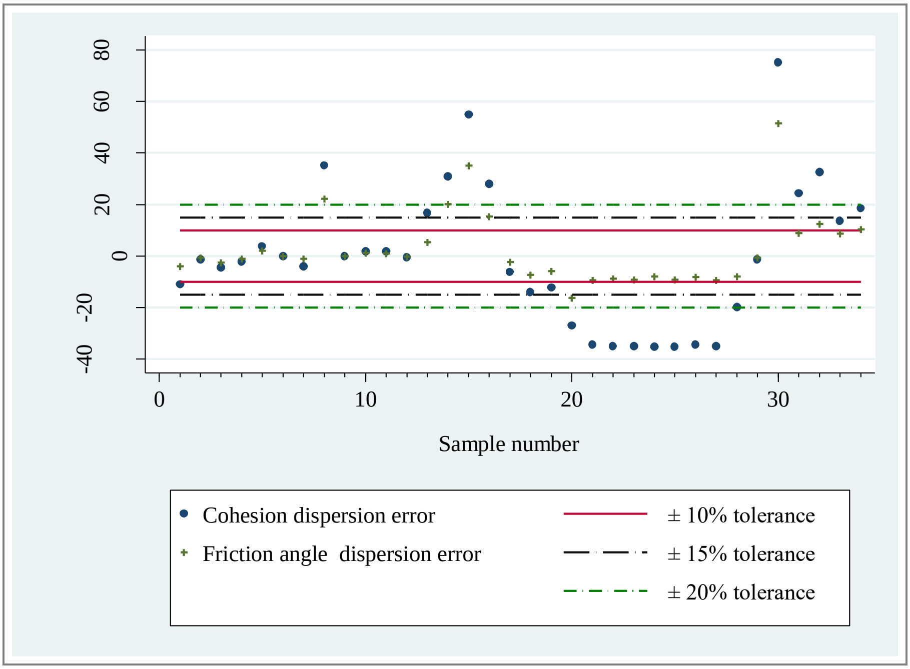


Fig. 5. Percentual error of cohesion and friction angle, when $20 \leq GSI' \leq 80$. Authors' own elaboration. Note. Software Stata 14.2 Windows software. College Station, Texas 77845 USA: StataCorp.

Meanwhile, the analysis of the failure criterion using *GSI′* provides approximate results for Tensile strength (σ_t); Uniaxial compressive strength (σ_c); Global strength (σ_{cm}); Modulus of deformation (E_{rm}). In this case (Fig. 6) it is observed that the parameters present dissimilar behaviors: there is great dispersion of the data, and a grouping of dispersed data stands out mainly between samples 28–34, which belong to the same geological origin (Ganerød et al. 2007) factor that also influences the estimates of *GSI′*, cohesion and friction angle. Despite this, Fig. 6b shows the data with a close-up, noting the scatter of the percentage error between +40% and –60%.

Thus, for the failure criterion it is obtained that: if an error tolerance of ±10%, ±15%, and ±20% is considered, it is obtained that the Tensile strength parameter, 18%, 20% and 31% of the estimates are within these tolerances, respectively; that Uniaxial compressive strength, 20%, 31%, 38% of the estimates are within these tolerances, respectively; that the Global strength parameter, 36% of the estimates are within these tolerances; that the Global strength parameter, 36%, 40%, and 51% of the estimates are in this tolerance range, respectively; that the Modulus of deformation parameter, 38%, 47%, and 51% of the estimates are in this tolerance range, respectively (Fig. 8). Despite the dispersion of the data, it is observed that the average of the errors made in the estimation of Global strength and Modulus of deformation is still low (–3.2% and 1.6%, respectively), very different from the estimations of Tensile strength and Uniaxial compressive strength, which greatly underestimate the parameter, making this method unreliable for these parameters (–36.2% and –23.9%, respectively).

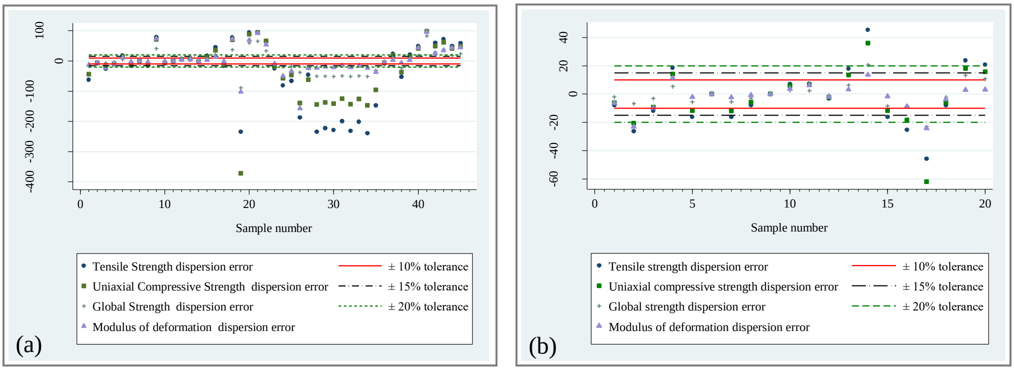


Fig. 6: Dispersion error: (a) porcentual error of rock mass parameters and (b) percentage error of rock mass parameters between –60% and 40%. Authors' own elaboration. Note. Software Stata 14.2 Windows software. College Station, Texas 77845 USA: StataCorp.

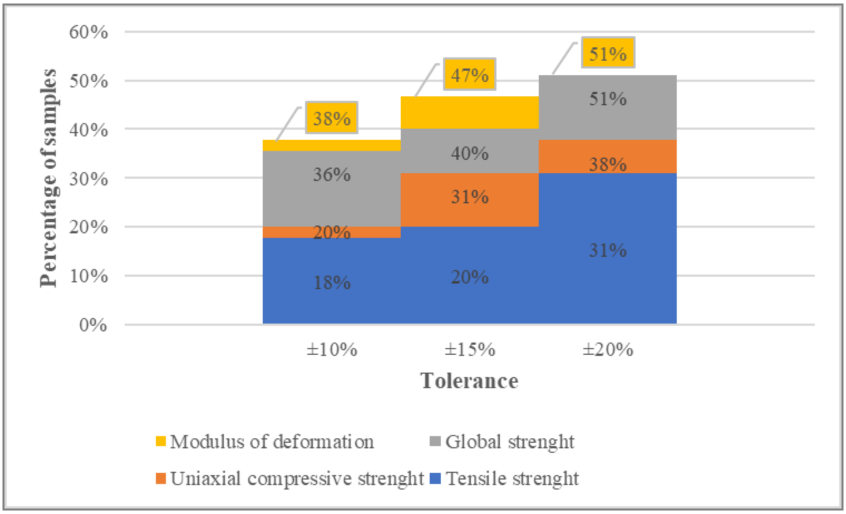


Fig. 7. Percentage amount of data as a function of a tolerance in the estimation error. Authors' own elaboration

As before, the use of *GSI′* is restricted according to the recommendation of the original authors (20 ≤ *GSI′* ≤ 80). Thus, for the Hoek–Brown failure criterion we obtain that: if an error tolerance of ±10%, ±15%, and ±20% is considered, it is obtained that the parameter Tensile strength, 16%, 18%, and 20% of the estimates are in those tolerances range, respectively; that Uniaxial compressive strength, 18%, 20%, 22% of the estimates are in those tolerances range respectively; that the Global strength parameter, 24%, 24%, and 33% of the estimates are in this tolerance range, respectively; that the Modulus of deformation parameter, 20%, 29%, and 33% of the estimates are in this tolerance range, respectively (Fig. 8). Despite the dispersion of the data, it is observed that the average of the errors made in the estimation of Global strength and Modulus of deformation is still low (–2.9% and 2.7%, respectively), very different from the estimations of Tensile strength and Uniaxial compressive strength, which greatly underestimate the parameter, making this method unreliable for these parameters (–43.8% and –37.1%, respectively, Fig. 8).

By taking this recommendation, it is observed that the number of estimates that would be in the tolerance intervals drastically decreases, again showing that the error is accumulating and that the failure criteria are greatly affected by the *GSI*, giving rise to a new question: why, if the failure criterion is so sensitive to the *GSI*, are we still using a parameter that depends so much on the observer's judgment and experience?

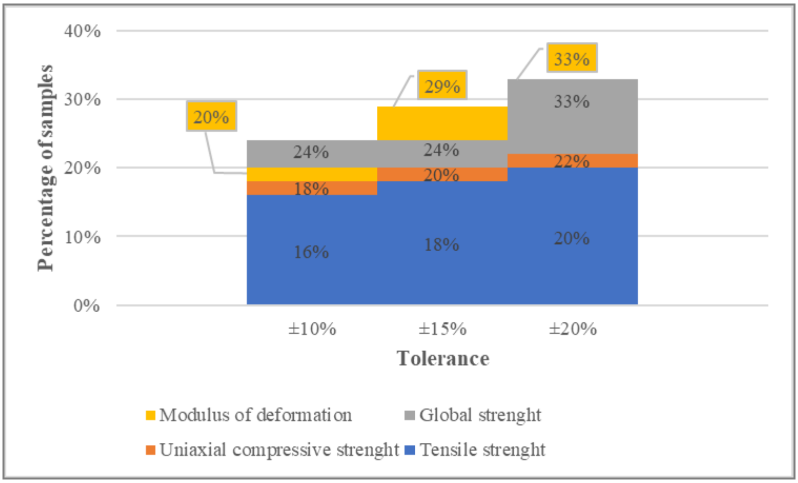


Fig. 81. Percentage amount of data as a function of tolerance in the estimation error, when 20 ≤ *GSI′* ≤ 80. Authors' own elaboration

4.1. ESTIMATION OF RESULT QUALITY

To evidence the closeness between the parameters determined by the failure criteria, first, without considering the empirical method and then, considering the *GSI′* empirical method, the linear correlation coefficient is used. If the correlation coefficient were equal to one, it would indicate that both the original value and the estimated values are equal, therefore, the more similar these values are, the higher the correlation coefficient can be expected.

4.1.1. ESTIMATION OF QUALITY OF THE MOHR–COULOMB CRITERION

For the case of cohesion in the Mohr–Coulomb Criterion, the correlation coefficient is R^2 = 0.9738 (without intercept), which indicates a very strong correlation, of a direct (positive) type, thus showing that it is possible to obtain a cohesion value from an empirical method of classification of geological characteristics of the rock (Fig. 9a).

Meanwhile, for the case of friction angle, the same statistical test shows that the correlation coefficient of R^2 = 0.9854 (without intercept) as shown in Fig. 9b. This indicates a very strong correlation, of direct type (positive), which shows that it is possible to obtain a friction angle value from an empirical method of classification of geological characteristics of the rock. In both cases, the estimated value for cohesion is very close to the original value determined by the Mohr–Coulomb failure criterion, so the linear adjustments obtain high correlation coefficients, and the average errors committed are close to zero, being: –1.1% the average error committed in cohesion; and –1.6% the error committed in the friction angle.

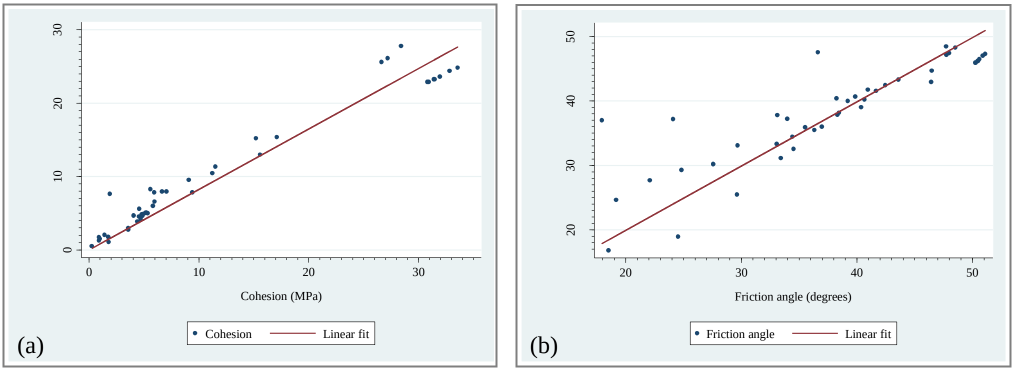


Fig. 9. Linear fit: (a) cohesion, (b) friction angle. Authors' own elaboration.
Note. Software Stata 14.2 Windows software. College Station, Texas 77845 USA: StataCorp.

4.1.2. ESTIMATION OF QUALITY OF THE HOEK–BROWN CRITERION

For the case of tensile strength in the Hoek–Brown Criterion, the correlation coefficient is R^2 = 0.6560 (without intercept), which indicates a moderate to strong, positive correlation (Fig. 10a); meanwhile, for the case of uniaxial compressive strength, performing the same statistical test shows that the correlation coefficient of R^2 = 0.8419, as observed in Fig. 9b, which indicates a very strong correlation, of direct type (positive). In both cases, although the correlation is good to strong, a large dispersion is noted, which is consistent with what is observed in Fig. 6.

Similarly, measuring the quality of the correlation for Global strength (Fig. 10c) and Modulus of deformation (Fig. 10d), the correlation coefficients are R^2 = 0.9615 (almost perfect positive) and R^2 = 0.9135 (positive, between strong and perfect). In these cases the dispersion is quite smaller, which is in agreement with the average error determined, and which causes a better reliability in this estimation compared to that observed in Figs. 10a and 10b.

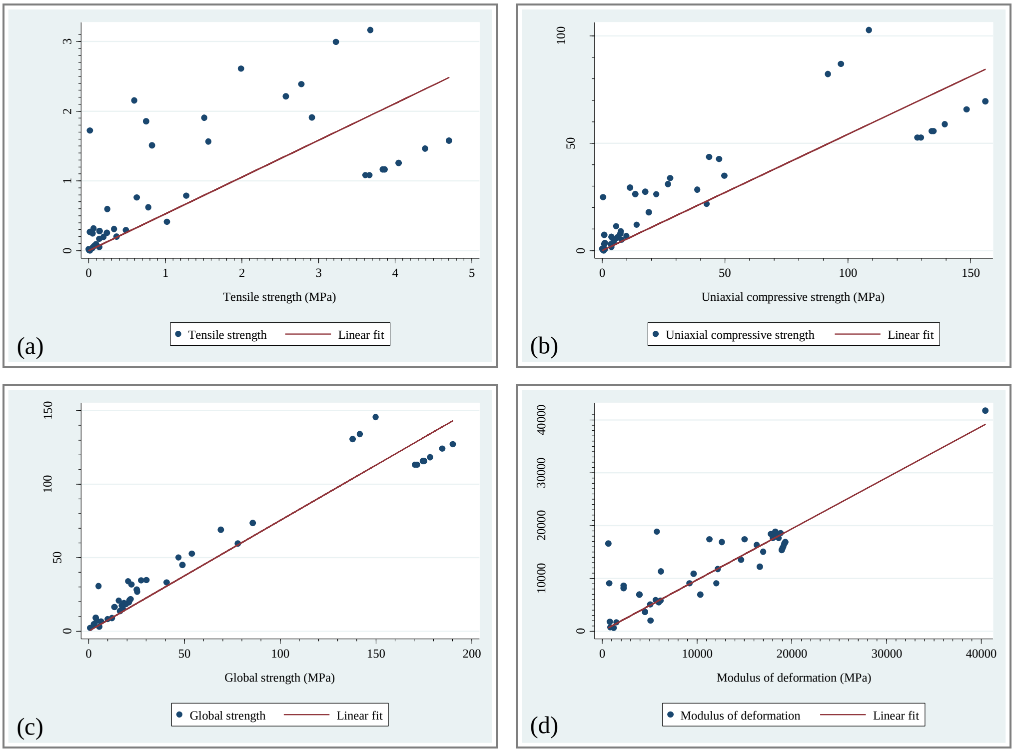


Fig. 10. Linear fit: (a) tensile strength, (b) uniaxial compressive strength, (c) global strength, (d) modulus of deformation. Authors' own elaboration.
Note. Software Stata 14.2 Windows software. College Station, Texas 77845 USA: StataCorp.

5. CONCLUSIONS AND RECOMMENDATIONS

1. In the analysis of cohesion according to the Coulomb criterion, a coefficient of correlation of R^2 = 0.9738 was obtained, indicating a very strong direct (positive) correlation. This suggests that it is feasible to infer a cohesion value using an empirical

method for classifying the geological characteristics of the rock. The analysis of the friction angle reveals a coefficient of correlation of $R^2 = 0.9854$. This result also indicates a very strong and direct (positive) correlation, demonstrating that it is possible to estimate the friction angle value using an empirical method for classifying geological characteristics.

In both cases, the estimated values for cohesion closely approximate the original values determined by the Coulomb failure criterion. The linear fits exhibit high coefficients of correlation, and the mean errors are close to zero, with a mean error of –1.1% for cohesion and –1.6% for the friction angle. Based on the results obtained, the use of the *GSI′* empirical method is recommended as input data in the Coulomb failure criterion to obtain approximate values of cohesion and friction angle.

Therefore, it is possible to estimate the Coulomb failure criterion with the empirical method studied, correctly estimating between 78% and 87% of the friction angles with a tolerance of between 10 and 20% respectively and between 38% and 62% of the cohesions under the same tolerance.

2. In the analysis of tensile strength according to the criterion, the correlation coefficient was observed to be $R^2 = 0.6560$, indicating a moderate to strong positive correlation. In contrast, the uniaxial compressive strength evaluation yielded a correlation coefficient of $R^2 = 0.8419$, demonstrating a very strong direct (positive) correlation. Although good to strong correlations were observed in both cases, significant scatter was observed, consistent with the observations in Fig. 6.

Furthermore, evaluation of the overall strength and deformation modulus revealed correlation coefficients of $R^2 = 0.9615$ and $R^2 = 0.9135$, respectively. These figures indicate an almost perfect positive correlation for the overall strength and a positive correlation ranging from strong to perfect for the deformation modulus, with considerably less scatter, consistent with the mean error determined. This increases the reliability of these estimates compared to those observed for tensile strength and uniaxial compressive strength.

In view of the above, the use of the empirical *GSI′* method in the nonlinear failure criterion is recommended only if an approximation to the values of overall strength and modulus of deformation is sought. However, the difficulty in obtaining accurate estimates of tensile strength and uniaxial compressive strength persists. Therefore, based on the evidence presented, it is not recommended to estimate these values; it is essential to perform their determination by standardized laboratory tests.

3. The evidence suggests that the empirical *GSI′* method is significantly more effective for estimating the Mohr–Coulomb criterion, whereas satisfactory results are not achieved when estimating the failure criterion. Therefore, it is recommended to use this predictive method solely during the conceptual engineering phase and as an initial guide within rock engineering projects, limited to the Mohr–Coulomb criterion.

4. The dispersions and errors observed in the estimates are attributable to both the amount of data sampled and their secondary origin. For this reason, it is imperative to adhere to the context of the recommendation given in this article whenever the empirical method is to be used in the application of the breakage criteria analyzed here.

In view of the above, it is recommended that an expanded database be used for future work, and that cross-validation techniques be implemented, with the intention of strengthening these results, thus extending the applicability of the method.

LIST OF SYMBOLS
(Authors' own elaboration)

Symbol	Description	Application
a	Constant that depends on the geomechanical characteristics of the rock mass, and on the Geological Strength Index.	Eq. (7)
D	Disturbance factor	Table 1
m_b	It is a value that is a function of: the material constant (m_i); the geologic resistance index; and the disturbance factor D	Eq. (5)
m_i	It is a constant that depends on the properties of the rock matrix	Eq. (5)
s	It is a value that is a function of the geologic resistance index and the disturbance factor D	Eq. (6)
σ_1	Major principal stress, used in Mohr–Coulomb failure criterion	Fig. 1b, Eq. (2)
σ_3	*M*inor principal stress, used in Mohr–Coulomb failure criterion	Fig. 1b, Eq. (2)
σ_c	Uniaxial compressive strength, used in the failure criterion	Fig. 2
σ_c	Uniaxial compressive stress, used in Mohr–Coulomb failure criterion	Fig. 1b, Eq. (2)
σ_{cm}	Global strength of the rock mass, used in the failure criterion	Fig. 2
σ_N	Normal stress, used in the Mohr–Coulomb failure criterion	Fig. 1a, Eq. (1)
σ_t	Tensile strength, used in the failure criterion	Fig. 2
ϕ	Angle of internal friction of the material, used in the Mohr–Coulomb failure criterion.	Fig. 1a, Eq. (1)
τ	Shear stress, used in the Mohr–Coulomb failure criterion	Fig. 1a, Eq. (1)

LIST OF ABBREVIATIONS

Symbol	Description	Application
E_{rm}	Modulus of deformation of the rock mass	Fig. 2
GSI	Geological Strength Index established by Hoek (1997)	Section 2.2
GSI′	Geological Strength Index established by Vivanco and Avendaño (2022)	Section 2.3
RQD%	Rock Quality Designation, established by Deere, D. U. (1964)	Eq. (8)

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